FREE STATION METHOD OF LEVELING

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ABSTRACT: A new method of leveling, suitable for high-precision industrial applications, is described. The observation procedure involves only minor modifications to the standard observation procedure for precise leveling. The analysis procedure is based on a simple linear least-squares adjustment model. Two applications of the method are given, one in which the usual type of precise leveling equipment is used and one in which modern electronic digital leveling equipment is used. Standard deviations of point elevations in these applications are on the order of 0.1 mm or greater.

INTRODUCTION

In high-precision industrial surveys [see Teskey (1998) for examples], determination of heights and height changes (vertical movements) is often very important, with required levels of precision typically on the order of 0.1 mm.

The industrial environment almost always presents difficult observing conditions [line of sight obstructions, poor lighting, high temperatures, temperature gradients, vibrations, etc. (Bayly and Teskey 1992)] that can produce small systematic errors. To achieve the previously noted levels of precision and to produce reliable estimates of heights and height changes, the effects of these small systematic errors must be detected and minimized. To accomplish this, a new method of precise leveling was developed. The method (which will be referred to as the free station method of leveling) and two applications of the method are described in the following sections.

OBSERVATION PROCEDURE

The observation procedure for free station leveling requires only two small modifications to the standard observation procedure for precise leveling, the modifications being that no attempt is made to balance backsight and foresight distances (free stationing) and that, at any given instrument setup, all points that can be observed are observed. The same equipment used for precise leveling is used for free station leveling. Use of this equipment is well-documented [e.g., Anderson and Mikhail (1998)], and a description of how it is used will therefore not be repeated here.

Because backsight and foresight distances are not balanced in free station leveling, the collimation error in the instrument (deviation of the instrument line of sight from the horizontal line, given as mm height/m horizontal distance) will produce an error in height difference between any pair of observed points. The error, however, can easily be corrected. The method of correction is described in the following section.

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ANALYSIS PROCEDURE

The analysis procedure is based on least-squares adjustment of the observed rod readings. The observation equation for free station leveling is expressed as

$$HI_i - (R_{ik} + CD_{ik}) = E_k$$

where HI_i = height of instrument at station i ($i = 1, \ldots, m$); R_{ik} = rod reading from instrument at station i to rod at point k ($k = 1, \ldots, n$); C = collimation error; D = horizontal distance from station i to point k; and E = elevation (height) of point k. Note in the above equation how each rod reading R_{ik} is corrected for the effect of the colimation error C by the term CD_{ik}.

With the observation equation given above, elevations of all points of interest E_k can be computed by standard parametric least-squares adjustment [see Anderson and Mikhail (1998) or Wolf and Ghilani (1997) for a detailed treatment of least-squares adjustment, including useful computer code in the latter reference].

Some important points related to least-squares adjustment of free station leveling based on the observation equation given above are as follows:

- The least-squares adjustment is linear, meaning that only one step (one iteration) is required to correct the approximate values of the unknown parameters (HI_i, C, and E_k) to their final least-squares adjusted values.
- Approximate values for HI_i and E_k (to begin the adjustment) can be computed from any subset of rod readings R_{ik}. A reasonable approximate value for collimation error C is zero. (Approximate values of zero could actually be used for all the unknown parameters. If these values were adopted the only consequence would be large corrections to the unknown parameters HI_i and E_k. The final adjusted values of all the unknown parameters would be the same, regardless of the approximate values chosen, because the adjustment is linear.)
- Horizontal distances D_{ik} can be measured by any convenient method. They need only be correct to the nearest cm.
- The adjustment should include computation of standardized residuals (also often referred to as data snooping values) to allow for the detection of blunders and systematic errors.

APPLICATIONS OF METHOD

Agrium Inc.

The method was first applied at the Agrium Inc. plant near Red Deer, Alberta, Canada, which produces anhydrous ammonia fertilizer from a methane gas feedstock. At this plant, a large electric motor and gearbox that drive a refrigeration compressor were experiencing very high vibration levels in response to operating load changes. One possible source of this problem was vertical movement in eight large reinforced concrete columns supporting the electric motor-gearbox-compressor unit. The actual source of the problem was eventually found to be cracks in welds inside the electric motor (Teskey et al. 1997); however, the results from free station leveling allowed column movement to be eliminated as a possible cause of the vibration problem at the beginning of the investigation. Only the results from the first epoch of free station leveling are presented in this section. Second epoch results were almost exactly the same, both in terms of elevations and associated standard deviations.

The free station leveling observations are shown in Fig. 1. A typical point mounted near the base of a reinforced column is shown in Fig. 2. Each of these points consists of a stainless steel plug permanently mounted in a concrete anchor. Observations were made with a Zeiss Ni 2 precise level and the standard precise leveling equipment. The results of the free station leveling are summarized in Table 1.

With an input standard deviation of 0.10 mm for each rod reading, the variance factor is 0.91. The number of degrees of freedom or total redundancy is 12 (25 rod readings less seven elevation unknowns less five heights of instrument unknowns less one collimation error unknown). The standard deviations of point elevations (except for point 1 which is fixed) are very homogeneous, varying only from 0.08 mm (80 μm) to 0.13 mm (130 μm).

None of the data snooping values indicate that the corresponding obser-

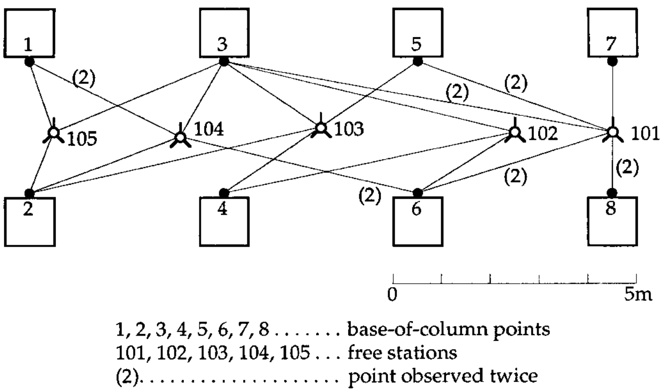

FIG. 1. Free Station Leveling Observations (Agrium Inc.)

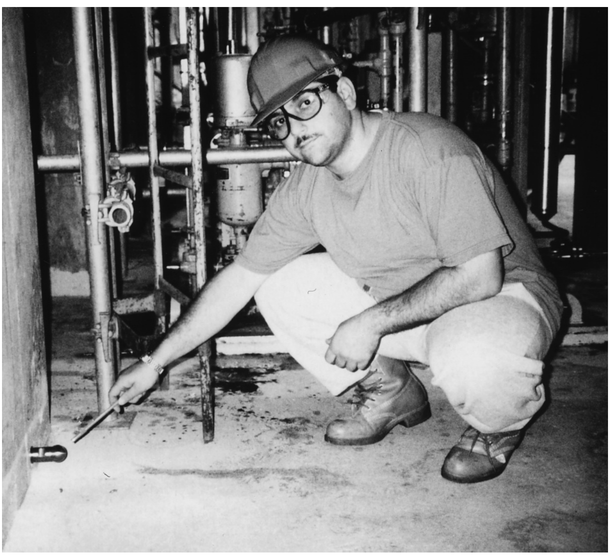

FIG. 2. Typical Base-of-Column Point

TABLE 1. Free Station Leveling Results (Agrium Inc.)

Point number (1)	Elevation (m) (2)	Standard deviation (mm) (3)
1	100.00000	0 (fixed point)
2	100.01315	0.09
3	99.98221	0.08
4	99.99986	0.11
5	100.00326	0.10
6	100.00889	0.09
7	100.02368	0.13
8	100.00471	0.11

vation could be rejected for inclusion of a blunder or systematic error. The average data snooping value is 0.79, with 18 in the range 0–1, 4 in the range 1–2, and 3 in the range 2–3 (largest value: 2.65). If any large data snooping values had been computed [values >3 could be flagged (Wolf and Ghilani 1997)], one of the following actions could have been taken:

- Delete the flagged observations and readjust. This is probably a feasible option in this application since all points (except for point 7) are connected by more than one observation.
- Reobserve and replace flagged observations, then readjust.
- Take corrective action to minimize a suspected systematic error [e.g., mix air with an industrial blower in the vicinity of the instrument if a refraction error is suspected (Teskey 1998)], reobserve and replace flagged observations, then readjust.

University of Calgary

The method was also applied at the University of Calgary Central Heating and Cooling Plant. This facility was chosen not because a problem existed but because it was a conveniently located industrial site.

In this application, a Topcon DL-102 electronic digital level was used, along with a bar code invar leveling rod. Points were set on the concrete floor so that the geometry of points 1–8 inclusive was exactly the same as that shown in Fig. 1. The points consisted of 9.5 mm (3/8 in.) inside diameter nuts epoxied to the floor. (These nuts were removed as soon as the leveling was completed.) A 12.7 mm (1/2 in.) diameter ball bearing was moved from point to point when observations were made.

As in the previous application, the number of degrees of freedom is 12, but the configuration of lines of observation is different than that shown in Fig. 1 due to obstructions that are almost always encountered in industrial surveys. (In this case, the main obstructions were two large cooling water pipes and a pressure vessel.) The other constraints on lines of observation were a minimum standoff distance of 2.3 m and low lighting on one line of observation. (The low lighting was flagged by the instrument and rectified by using a flashlight to illuminate the rod.)

Standard deviations of point elevations are shown in Table 2. Again, they are very homogeneous, varying only from 0.04 mm (40 μm) to 0.07 mm (70 μm). None of the data snooping values are >3.

TABLE 2. Free Station Leveling Results (University of Calgary)

Point number (1)	Elevation (m) (2)	Standard deviation (mm) (3)
1	100.00000	0 (fixed point)
2	100.00699	0.07
3	99.99144	0.05
4	99.99656	0.05
5	100.01243	0.07
6	100.01225	0.05
7	99.99960	0.05
8	99.99131	0.04

CONCLUSIONS

Free station leveling is a special method that can be applied in industrial surveys where the required level of precision is on the order of 0.1 mm or greater. The method works well with the usual type of precise leveling equipment or modern electronic digital leveling equipment.

If electronic digital leveling equipment is used, the method is more convenient and faster for two reasons: distance is measured electronically rather than by stadia or tape; and all data are stored electronically. For the two applications described in this paper, the use of electronic digital leveling equipment produced better results for very similar configurations of lines of observation; this, however, many not always be the case since the observing conditions were different in the two applications.

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